Persistent Visitation

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Introduction

Motivation

Persistent Intelligence, Surveillance, and Reconnaissance missions

- Objects of interest
  - Time varying properties
- Unmanned aerial vehicles
  - Equipped with sensors
  - Possible fuel constraints
- Friendly bases
  - Can be used as fuel depots
  - May require assistance in protection
    - Tracking of nearby opposing forces
Global Objectives

- Generate paths for UAVs such that persistent ISR objectives are met:
  - Inference of object of interest properties.
  - Maintenance of UAV fuel.
  - Tracking of opposing forces in theatre of operations.

Year 1 (2011-2012)

- Single UAV with or without fuel constraints monitoring properties of objects of interest.
- Multiple UAVs with or without fuel constraints monitoring properties of objects of interest and tracking an intruder close to base of operations.
Many manned and unmanned Air Force platforms perform persistent ISR missions.

These platforms collect large amounts of data which require many operators to analyze before classification decisions can be made and action taken.

Automating part of this data analysis to reduce human workload is a priority.

These platforms can also assist with communications if need be:

"Persistent near-space systems, potentially in the form of ultra-long endurance airships or autonomous flight vehicles, can ensure theater-level communications relay functions in the event of loss or degradation of corresponding space systems."

- Technology Horizons Report
Outline

- Literature review
  - Original contributions
- Persistent Visitation with or without Fuel Constraints
  - Model & Dynamics
  - Periodicity of Paths
  - Tours
  - Cost minimization
- Persistent Visitation, Detection, and Capture
  - Model & Dynamics
  - Intruder tracking
  - UAV Path Selection
- Conclusion & Future work
**Introduction**

**Literature review**

- **Persistent Coverage for Vehicles**
  - On persistent coverage control. [Hokayem et al. '07]
  - An optimal control approach for the persistent monitoring problem. [Cassandras et al. '11]

- **Patrolling**
  - Developing a deterministic patrolling strategy for security agents. [Basilico et al. '09]
  - Persistent patrol with limited range on-board sensors. [Huynh et al. '10]

- **Routing Problems with Fuel Considerations**
  - Vehicle Routing Problem for Emissions Minimization [Figliozzi '10]
  - To Fill or not to Fill: The Gas Station Problem [Khuller et al. '11]

- **Intruder Capture**
  - Patrolling security games: Definition and algorithms for solving large instances with single patroller and single intruder [Basilico et al. 12]
Original contributions

i. We formulate the **persistent visitation problem** with no a priori assumptions on periodicity for a single UAV.

ii. We prove the **existence of periodic solutions** to the problem.

iii. We present a **complete algorithm** to solve for the periodic path with minimal cost.

iv. We formulate the **persistent visitation, detection, and capture problem** for multiple UAVs.

v. We present an **algorithm** to solve for UAV paths to maximize likelihood of imaging an intruder and minimize the likelihood of not monitoring properties of objects of interest.
Model

**Single UAV**

- Constant velocity: $v$, Fuel capacity: $F$
- Fuel consumption rate: $\dot{f}_c$

$n$ customers

- Revisit deadline: $r_i$

$q$ fuel stations

- Cost (per unit fuel): $c_j$

Area modeled as graph

- Graph $G = (N, E)$, $N = \{ \{ n \text{ customers} \} \cup \{ q \text{ fuel stations} \} \}$
- Distance: $d_{i,j}$, $i, j \in N$
## Model

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Key assumptions

- Visit and refueling durations negligible in comparison to transit time.

States

- Location being visited at step $k$: $p(k)$
- Total time elapsed at step $k$: $\tau(k)$
- Discrete event states: $y(k) = [p(k) \quad \tau(k)]^T$
- Slack time of destination $i$: $x_i(t)$
- Amount of fuel carried: $f(t)$
- Mission state (continuous time): $m(t) = [x_1(t) \quad \cdots \quad x_n(t) \quad f(t)]^T$

Inputs

- Next location to be visited: $u(k, 1)$
- Amount refueled if next visit is to a depot: $u(k, 2)$
Discrete event dynamics

\[ p(k + 1) = u(k, 1) \]  
\[ \tau(k + 1) = \tau(k) + \frac{d_p(k), u(k, 1)}{v} \]

Continuous time dynamics

\[ \dot{x}_i(t) = -1 + \sum_{j=1}^{k} \delta_{ip(j)} \cdot (r_i - x_i(\tau(j))) \cdot \delta(t - \tau(j)), t \leq \tau(k) \]

\[ \dot{f}(t) = -\dot{f}_c + \sum_{j=1}^{k} u(j, 2) \cdot \delta(t - \tau(j)), t \leq \tau(k) \]
Problem

Constraints

- Time between consecutive visits to destination \( i \) must be less than or equal to \( r_i \) ⇒
  \[
  0 \leq x_i(t)
  \]
  (5)
- Amount of fuel carried must be positive and less than the fuel capacity ⇒
  \[
  0 \leq f(t) \leq F
  \]
  (6)

Persistent Visitation Problem

- Find an infinite sequence of visitations that does not violate the stated constraints and minimizes cost.

Existence of Solutions

- Problem configurations that are unsolvable exist.
Heuristics

We use heuristics when the UAV does not need to refuel ($\dot{f}_c = 0$). Since there is no refueling, the cost aspect of the problem is removed.

**Reactive Heuristics**
- Earliest Deadline First

**Search Ahead Heuristics**
- Maximum Minimum Slack Time
- Maximum Sum of Slack Times

**Completeness**
- Proved by counter-example that Earliest Deadline First is incomplete.
- Search ahead heuristics are incomplete for search horizons $< \max(r_i)$.
- Search ahead heuristics are complete for search horizons $\geq \max(r_i)$. 
Figure: Layout of objects of interest.
Simulations with Heuristics

Object slack times and visits over time using the earliest deadline first heuristic

Object slack times and visits over time using the two step maximum sum of slack times heuristic

Earliest Deadline First Example

Two Step Maximum Sum of Slack Times Example
Number of successes for heuristics in Monte Carlo simulations with varying number of objects of interest

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Existence of periodic solutions

**Path**
An infinite sequence of visitations such that the constraints are not violated.

**Tour**
A sequence of visitations starting from one destination and ending at the same destination such that all other destinations have been visited.

- A path can be viewed as an infinite sequence of tours.
- The initial mission state of these tours belongs to a compact set.
- **Theorem**: If a path exists, then a periodic path also exists.
Minimizing cost when refueling

Method

- Find all tours that satisfy slack time constraints and can satisfy fuel constraints without solving for refuel amounts.
- Solve a constrained minimization problem for each tour to calculate the amounts to refuel which minimize total cost during tour.
- Select tour with minimum cost divided by tour duration, thus generating a path with the minimized cost per unit time.
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Finding tours

- A tour that solves the problem will need to satisfy the given set of constraints below.

### Continuity constraints
- Time between consecutive visits to destination $i$ must be $\leq r_i$.
- Time between consecutive visits to a fuel station must be less than or equal to the endurance of the vehicle.

### Closure constraints
- Time between last visit to destination $i$ and first visit to destination $i$ must be $\leq r_i$.
- Time between last fuel depot visit and first fuel depot visit must be less than or equal to the endurance of the vehicle.

- Perform full breadth-first tree search to find tours that satisfy constraints.
Calculating amounts of fuel to be purchased

- Define sequence of depot visits for a tour: \( W: \{w_1, w_2, ..., w_m\} \).
- \( \Delta t_i \) is the time in transit between \( w_i \) and \( w_{i+1} \).
- \( \Delta f_i \) is the amount refueled when visiting \( w_i \).

Amount refueled must be large enough to reach next depot but small enough as to not surpass the fuel capacity.

\[
\Rightarrow \dot{f}_c \sum_{j=1}^{i} \Delta t_j \leq \sum_{j=1}^{i} \Delta f_j \leq F + \dot{f}_c \sum_{j=1}^{i-1} \Delta t_j , \ 1 \leq i \leq m \quad (7)
\]
Calculating refueling amounts (Cont’d)

Solve constrained minimization problem

\[
\dot{f}_c \sum_{j=1}^{i} \Delta t_j \leq \sum_{j=1}^{i} \Delta f_j \leq F + \dot{f}_c \sum_{j=1}^{i-1} \Delta t_j, \quad 1 \leq i \leq m
\]

\[
0 \leq \Delta f_i \leq F
\]  \hspace{1cm} (8)

\[
\min_{\Delta f_i, 1 \leq i \leq m} \left\{ \sum_{j=1}^{m} \Delta f_j \cdot c_{w_j} \right\}
\]  \hspace{1cm} (9)
Select best tour

Select tour with minimum cost divided by tour duration.

\[
T_{best} = \min_{T \in \Pi} \left\{ \frac{\sum_{j=1}^{m} \Delta f_j \cdot c_{w_j}}{L(T)} \right\}
\]  

Generate path with minimized cost per unit time.
Algorithm Performance

Completeness
- Algorithm is complete when tours start at customers
  - Length of tour is constrained by the revisit deadline of the first customer visited $\Rightarrow$ finite number of tours

Complexity
- The algorithm's worst-case complexity is:
  $O((n + q - 1)^{\frac{v \cdot \max(r_i)}{\min(d_{i,j})}})$
Persistent Visitation and Capture
Introduction

Base Defense Scenario

Base located on a road network and UAVs are tasked with imaging approaching intruders

$n$ Unattended Ground Sensors placed at road intersections
- Sensors keep track of last time a vehicle passed its intersection.
- Sensors cannot communicate with each other.
- Each sensor has a revisit deadline set by the mission designer.
- Revisit deadlines reflect the priority of visiting certain sensors.
- Cartesian coordinates: \((\xi_i, \zeta_i), 1 \leq i \leq n\).

1 Base/Fuel Depot
- Cartesian coordinates: \((\xi_{n+1}, \zeta_{n+1})\).

Area modeled as a graph \(G = (N, E)\)
- \(N = \{\text{n sensors}\} \cup \{\text{base}\}\)
- \(E = \{\text{roads}\}\)
Figure: Map of base and road network.
### Intruder’s goal
- Reach the base.

### UAVs’ goal
- Minimize amount of time by which revisit to sensors are overdue.
- Maximize probability of imaging an intruder before they reach the base.
- Maintain positive amounts of fuel.

Intruder imaging occurs when UAV and intruder are collocated at an UGS
- Intruder velocity is variable $\Rightarrow$ UAVs need to wait at nodes to capture an intruder.
Intruder Characteristics

Velocity

- Velocity can take values between $V_{min}$ and $V_{max}$ as dictated by the probability distribution $V$:
  \[ V = \mathcal{N}(\mu_v, \sigma_v^2) \]  
  (11)

- Velocity distribution uniform across all edges in the graph.

Fuel

- Consumes minimal fuel $\Rightarrow$ model as having infinite fuel.

Decision Making

- From a node, an intruder selects the next node from a subset of $\gamma$ of the connected nodes with shortest paths to the base. The selection is made using a uniform distribution.

- The lower $\gamma$ is, the more aggressive the intruder is.
Intruder Characteristics (Cont’d)

Distance traveled by intruder modeled as a Lévy process

- **Lévy Process Definition:**
  - A process \( X = \{X_t : t \geq 0\} \) defined s.t.
    - Paths of \( X \) are almost surely right continuous with left limits.
    - \( \mathbb{P}(X_0=0)=1 \).
    - \( X_t - X_z = X_{t-z}, t > z \geq 0 \) (stationary increments).
    - \( X_t - X_z \) is independent of \( \{X_u : u \leq s\} \) (independent increments).

- **Distance traveled by intruder:** \( X_t \)

\[
X_0 = 0 \tag{12}
\]

\[
X_t - X_z = \mathcal{N}(\mu_v \cdot (t - z), \sigma_v^2 \cdot (t - z)^2), t > z \geq 0 \tag{13}
\]
Probability of intruder passing an UGS

- Given a path $s$ of nodes where $s(g)$ indicates the $g^{th}$ node visited and the fact that the intruder passed $s(1)$ at time 0.
- Probability of intruder passing node $s(2)$ at time $t$ is:
  \[
P(node_{\text{passed}} = s(2), s, t) = P[X_t = d_{s(1),s(2)}]
  \]  

\[
\downarrow
\]

- Probability of intruder passing node $j$ between times $t_i$ and $t_f$ before reaching the base given set of possible sequences $S$:
  \[
P(node_{\text{passed}} = j, S, t_i, t_f) = \sum_{a=1}^{\text{size}(S)} P(S(a)) \sum_{b=1}^{\text{length}(S(a))-1} (1 - \mathbf{1}_{S(a,1:b)}(n + 1))
  \]
  \[
  \delta_{S(a,b+1),j} \int_{t_i}^{t_f} P[X_\rho = \sum_{f=1}^{b} d_{S(a,f),S(a,f+1)}] d\rho
  \]

\[
(15)
\]
Find a path for the UAVs within a given time horizon, $t_{\text{horizon}}$, that minimizes a weighted sum of the amount of time by which revisits to sensors are overdue and the probability of not imaging the intruder before it reaches the base all while maintaining positive amounts of fuel.
Problem Formulation (Cont’d)

- **A**: cost per unit time of missing a deadline
- **B**: cost of not capturing an intruder
- **C**(u, y, l): cost of UAV actions, u, taken at step l from state y:

\[
C(u, y, l) = A\left(\sum_{i=1}^{n} \frac{\left| x_i(\tau(l + 1)) \right| - x_i(\tau(l + 1))}{2}\right) + B(1 - p_{\text{intruder capture before base visit}}(S(t_E, \tau(l + 1)), y(l), y(l + 1)))
\]  

(16)

**Problem**: Find a sequence u such that:

\[
\tau^{-1}(\lfloor \tau(k) + t_{\text{horizon}} \rfloor) \sum_{l=k}^{\tau^{-1}(\lfloor \tau(k) + t_{\text{horizon}} \rfloor)} C(u, y, l) \text{ is minimized}
\]  

(17)

\[
\forall t, f_j(t) \geq 0, 1 \leq j \leq m
\]  

(18)
Decision Making for UAVs

- Current system (UAV/slack times) state
- Time horizon for decisions
- Most recent intruder observation
- Maximum intruder velocity
- Intruder possible node transitions
- Propagate UAV/slack time dynamics forward
- UAVs velocity
- Possible actions for UAVs
- Possible intruder paths
- Propagate intruder dynamics forward

Las Fargeas et al. (UM & AFRL)
Persistent Visitation and Capture
September 19th, 2012
Decision Making for UAVs (Cont’d)

- Possible actions for UAVs
- Possible intruder paths
- Intruder velocity distribution

Formula for probability of intruder passing a given node in a given time window before reaching base

- Intruder capture probability
- Cost function
- Missed deadlines
- Cost of action for UAVs

UAV/slack time dynamics

Action for UAVs

Best action for UAVs

Cost minimization loop
Formulated the **persistent visitation problem** with no a priori assumptions on periodicity for a single UAV.

Proved the **existence of periodic solutions** to the problem.

Presented a **complete algorithm** to solve for the periodic path with minimal cost.

Formulated the **persistent visitation, detection, and capture problem** for multiple UAVs.

Presented an **algorithm** to solve for UAV paths to maximize likelihood of imaging an intruder and minimize the likelihood of not monitoring properties of objects of interest.
Future Work

- **Persistent Visitation**
  - Use of soft deadlines.
  - Inclusion of deterministic or stochastic task times.

- **Persistent Visitation, Detection, and Capture**
  - Allow UAVs to select wait time.
  - Allow UAVs to change destination in flight.
  - Implement multiple intruders in network.
  - Further investigation into problem computational complexity.

**Relevant publications**
