Aerothermoelastic Modeling for Hypersonic Vehicle Simulation

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Overview

• Overview of HSV ATE Simulation
• ROM Developments
  – Unsteady aero ROM
  – Thermo-elastic ROM
• Integrated Simulation through Partitioned Solution
• List of Publications
Aerothermoelastic 6DOF HSV Framework

Thermal
- **Fundamental:** N/A
- **ROM:** POD
- **High fidelity:** FEA
- **Inputs:** Heat flux
- **Outputs:** Nodal/wall temps

Elastic
- **Fundamental:** Beam modes
- **ROM:** Ritz modes
- **High fidelity:** FEA
- **Inputs:** Temps, aero loads, interface motion
- **Outputs:** Modal motion, interface loads

6DOF Model
- **Nonlinear EOMs**
- **Inputs:** Interface loads, control inputs
- **Outputs:** Body motion

Aeroheating
- **Fundamental:** Eckert ref. temp.
- **ROM:** Kriging
- **High fidelity:** CFD
- **Inputs:** Wall temps, flow properties
- **Outputs:** Heat flux

Unsteady Aero
- **Fundamental:** Piston theory
- **ROM:** Correction factor ROM
- **High fidelity:** CFD
- **Inputs:** Mode shapes, modal motion
- **Outputs:** Aero loads

Propulsion
- **Fundamental:** Quasi 1D flow
- **ROM:** MASIV
- **High fidelity:** CFD
- **Inputs:** Vehicle states, inlet flow conditions
- **Outputs:** Thrust, underbody pressures
Issues Addressed in ATE Research

• Previous (AFRL) Model Features:
  – 2-D longitudinal flight dynamics
  – Rigid control surfaces
  – Flexible states modeled using a beam (transverse direction only)
  – Thermo-elastic structure modeled within a beam
  – 1-D heating effects (through thickness) for material property degradation
  – Aerodynamics based on classical 2-D approximations or CFD table look-up

• Enhanced (MAX) Model Features:
  – 6 DOF flight dynamics
  – Flexible control surfaces
  – Flexible states modeled using a 3-D beam or general modal representation
  – Representative thermo-elastic 3-D structure
  – 3-D heating effects for material property degradation and thermal stresses
  – 3-D unsteady aerodynamics
Overview of ROM Generation Procedure

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Unsteady Aerodynamics Modeling

Key Contributions/Conclusions

• Correction factor ROM methodology successfully applied to calculation of lift, drag, and moment coefficients to a wide range of Mach numbers
  – Same mathematical form from takeoff to cruise, and then landing flight segments
• ROM errors in super/hypersonic regime are generally smallest
• ROM errors generally increase with reduced frequency, as shown by the facts that errors increase with oscillation frequency and inversely with Mach number
• Use of simplified models greatly assists in ROM construction by helping determine where/how many ROM sampling points are necessary
• Several potential methods exist for situations where $M_{sim} \neq M_{step}$, each of which produced relatively good results
• Upon construction, ROM for simple cases runs over two orders of magnitude faster than full-order solutions; efficiency will only increase with more complex CFD models
Overall Process for Reduced-Order Modeling of Unsteady Aerodynamics Across Multiple Mach Regimes

Aeroelastic Simulation Framework

Arbitrary Geometry
Thermoelastic structural analysis

Structural Mode Shapes
Modal Amplitudes

CFD Modal Step Input Runs
Linear Convolution

Nonlinear Correction Factor

Correction Factor CFD Runs

CFD runs used for model construction

Time-accurate coefficients and/or GAFs

ROM construction

Outputs

Inputs
Convolutions

- Response $y_{\text{conv}}(t)$ of a linear system to an arbitrary input $f(t)$ can be found using Duhamel’s integral:

$$y_{\text{conv}}(t) = f(0)H(t) + \int_{0}^{t} \frac{df}{dt}(\tau)H(t-\tau)\,d\tau$$

where $H(t)$ is the system’s unit step response.

- Duhamel’s integral may also be written in terms of system’s unit impulse response, but step response is used here due to ease of implementation into CFD code.

- $y_{\text{conv}}(t) \rightarrow$ linear, uncorrected ROM.

- Unit step response for each individual elastic mode shape found using CFD.

**Errors shown to increase for amplitudes much larger than step amplitude; nonlinear correction factor introduced to fix this.**
Nonlinear Correction Factor

• Nonlinear correction factor $f_c$ introduced to account for aerodynamic nonlinearity:

$$f_c = \frac{y_{nonlin}}{y_{lin}} + \delta$$

- $y_{lin}$: offset introduced to prevent numerical singularity
- $y_{nonlin}$: response of system after elastic modal deformations applied simultaneously (nonlinear in general)

• For linear system, $f_c = 1$ due to linear superposition; thus, ratio is measure of system’s nonlinearity

- Corrected response $y_{corr}$ can be found:

$$y_{corr} = f_c \left( y_{conv} + \delta \right) - \delta$$

Uncorrected ROM response from linear convolution

Challenge is to find value of correction factor over entire range of parameters: modal amplitudes, Mach number

Calculate quasi-steady response $\rightarrow y_{nonlin}$
Correction Factor CFD Simulations

Parameter space: modal amplitudes, flight conditions

Parameter space sampling points

Steady CFD runs

Kriging surfaces

- Method to create representation of objective function based on results of certain number of function evaluations
- Takes advantage of lack of random error in computer experiments

\[ f_c(\eta_1, \eta_2, \ldots, \eta_m, M) = \frac{y_{\text{nonlin}} + \delta}{y_{\text{lin}} + \delta} \]

- Modal amplitudes applied
- Steady coefficient values found
- Correction factor calculated

Response to steady CFD run

\[ y_{\text{nonlin}} \]
Sampling Point Determination

Need efficient way to distribute sampling points in parameter space

- Too few points → potential loss of accuracy
- Too many points → unnecessary computational expense

Employ computationally-efficient simplified models to aid in sampling point determination:

- Calculate \( f_c \) values using simplified model

Places points in levels throughout parameter space

Construct kriging surface

Stopping criterion met?

YES

Final sampling points

NO

Place additional point at location of max. MSE on surface

Simplified model for efficiency
Simplified Model Development

- **Challenge**: need method to overcome computational expense of calculating many individual sampling points with CFD
- **Basic idea**: elastically deformed wing can be thought of as series of chordwise-rigid segments along span which see different localized angles of attack
- Method of segments (MoS) uses limited number of steady, rigid CFD runs to efficiently generate aerodynamic coefficients (and correction factor values) for wide range of elastically deformed configurations
- Captures 3-D effects, unlike pure strip theory
Method of Segments Process

Parameter space: \( M, \alpha \) → Steady CFD runs at various \( M, \alpha \) → Kriging surfaces for lift and drag forces on each segment over range of \( M \) and \( \alpha \)

Old parameter space: \( M, \eta_1, \eta_2, \ldots, \eta_m \)

Performed only once up front

Divide wing into segments

Steady CFD runs at various \( M, \alpha \)

Track \( L, D \)

Kriging surfaces for lift and drag forces on each segment over range of \( M \) and \( \alpha \)

Arbitrary modal displacement

Calculate local \( \alpha \) at each segment

Use \( M \) and local \( \alpha \) to find lift and drag forces on each segment

Calculate \( C_l, C_d, \) and \( C_m \) for entire wing

Calculate correction factor

No longer 1-to-1 correspondence between sampling points and CFD runs; \# sampling points >> \# CFD runs
ROM Testing Overview

- Euler solutions obtained using CFL3D code from NASA Langley
  - *Structured solver which can find Euler and Navier-Stokes solutions*
  - *Capable of finding unsteady solutions with mesh deformations*
- Accuracy of ROM tested by comparing with full CFD simulations
- Test case geometry:
  AGARD 445.6 wing
  - *Wing has been popular choice for aeroelastic studies*
  - *First 3 mode shapes considered*

Step sizes correspond to tip deflection of around 0.1% of span
Results Overview

- Simulations conducted from subsonic through supersonic regimes
- Items assessed through results:
  1. Applicability of ROM across wider range of conditions
  2. Effect of simulation Mach number ($M_{\text{sim}}$) moving away from step response Mach number ($M_{\text{step}}$) used for convolution
  3. Errors as function of number of sampling points ($N_{\text{samp}}$)
  4. Viability of Method of Segments over Mach regimes

- Assessment accomplished by taking 25 sets of modal parameters and conducting simulations at different Mach numbers:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (sub/super)</td>
<td>0.3/1.1</td>
<td>0.9/3.0</td>
</tr>
<tr>
<td>$d_1$</td>
<td>-50</td>
<td>50</td>
</tr>
<tr>
<td>$d_2$</td>
<td>-35</td>
<td>35</td>
</tr>
<tr>
<td>$d_3$</td>
<td>-30</td>
<td>30</td>
</tr>
<tr>
<td>$\omega_1$ (rad/s)</td>
<td>63</td>
<td>350</td>
</tr>
<tr>
<td>$K$</td>
<td>0.02</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Error metric:

$$L_1 \text{ error } = \frac{1}{N} \sum_{i=1}^{N} \left( \left| y_{\text{ROM},i} - y_{\text{CFD},i} \right| \right)$$

$$\max \left( y_{\text{CFD}} \right) - \min \left( y_{\text{CFD}} \right)$$

where:

- $N$ = number of time steps
- $y_{\text{ROM},i}$ = ROM response value at $i^{th}$ time step
- $y_{\text{CFD},i}$ = CFD response value at $i^{th}$ time step
Drag Coefficient Results: All $M_{\text{step}}$ values

- All subsonic/supersonic $M_{\text{step}}$ values included, not just $M_{\text{step}} = M_{\text{sim}}$
- Results similar to before: errors generally lower for higher Mach numbers

Data point $\rightarrow$ mean error over 25 simulations
Error bars $\rightarrow$ error standard deviation

Need to explore errors/trends for other coefficients
Lift Coefficient Results: All $M_{\text{step}}$ values

- Errors generally smaller and show different trend than $C_d$
- Errors lower for $M_{\text{sim}}$ values in middle of Mach ranges → closer to more values of $M_{\text{step}}$
- Suggests that lift coefficient is more sensitive to $M_{\text{sim}} - M_{\text{step}}$ separation than drag coefficient

In general, ROM methodology shown to agree well with CFD results over sub- and supersonic Mach ranges tested
Sampling Point Errors: Drag Coefficient

• ROMs constructed with varying number of kriging surface sampling points → additional points added at location of max. surface error
• In general, addition of extra points does not significantly reduce error, suggesting that sufficient number of points has been reached

Need to find out if these trends hold for other coefficients
Sampling Point Errors: Lift Coefficient

- Subsonic errors show no sensitivity to $N_{samp}$ at all for $N_{samp} > 100$
- Slight error decrease for some $M_{sim}$ values in supersonic range as $N_{samp}$ increases; eventually levels off
- Again shows that sufficient number of sampling points has been reached

Results show that errors observed are not due to insufficient amount of kriging surface sampling points
Example Test Case

- Practical example case over entire range of Mach numbers considered (Mach 0.3-3.0)
- Same simulation test cases as before: simulations of 25 sets of modal parameters conducted at various Mach numbers
- Suppose limited number of $M_{\text{step}}$ values are available → practically speaking, not going to have unlimited $M_{\text{step}}$ values to choose from
- Need method to pick which $M_{\text{step}}$ value to use for ROM response or to blend responses from more than one value of $M_{\text{step}}$
- Two separate methods have been introduced for this purpose
Methods for $M_{\text{step}}$ Selection

Suppose $M_{\text{sim}}$ is between two $M_{\text{step}}$ values, $M_{\text{step1}}$ and $M_{\text{step2}}$:

$$y_{Method2} = \begin{cases} y_{ROM,M_{\text{step1}}} & L_1 < L_2 \\ y_{ROM,M_{\text{step2}}} & L_1 > L_2 \end{cases}$$
Drag Coefficient Results

Original $M_{\text{step}}$ values

Addition of $M_{\text{step}} = 1.7$

- Each method performs comparably, showing that results are not overly sensitive to $M_{\text{step}} - M_{\text{sim}}$ separation
- Addition of $M_{\text{step}} = 1.7$ does not have significant effect on results
- Small error spike seen in transonic regime

Drag coefficient shown to be relatively insensitive to $M_{\text{sim}} - M_{\text{step}}$ separation
Lift Coefficient Results

**Original $M_{\text{step}}$ values**

- Weighted averages of Method 1 show significant improvement over Method 2 in subsonic regime.

**Addition of $M_{\text{step}}=1.7$**

- Unlike drag coefficient, addition of $M_{\text{step}}=1.7$ significantly improves results for both methods.

*Ideal choice of method depends on number of $M_{\text{step}}$ values available, range of Mach numbers, etc.*
Computational Efficiency

- Important to quantify computational savings of ROM over full-order solutions
- ROM results display time to compute response per coefficient
- Efficiency increase even greater for more complex geometries, as CFD time will increase while ROM time will not
- Also possible to decrease ROM time by finding more efficient computer coding algorithms

Each data point is mean CPU time over 25 simulations at a given Mach number
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**Aerothermoelastic Modeling**

**Key Contributions/Conclusions**

- Developed reduced-order transient thermal solution based on POD that is *robust* under time-varying boundary conditions
  - Fixed basis provides sufficient accuracy under range of flight conditions

- Created a novel structural dynamics representation under transient heating
  - Fixed Ritz-based modal solution augmented by heated static modes able to capture both highly dynamic and quasi-steady thermoelastic response
  - Kriging models allow for direct update of stiffness/thermal loads as a function of thermal POD modal coordinates
  - Retain accuracy under material property degradation and geometric stiffening
  - Capture both highly dynamic and quasi-static response with fixed basis
  - Employ small subset of modes while retaining sufficient accuracy

- Developed aerothermoelastic framework consisting of *dissimilar* reduced-order transient thermal, structural dynamic, and aerothermal models
  - Computational cost significantly reduced compared with full-order solution
  - From numerical studies:
    - Aerothermoelasticity impacts control surface effectiveness
    - Control surface inertia results in significant variation in instantaneous forces and hinge moments under excitation
Aerothermoelastic ROM Framework

- Update Heat Flux
- Thermal BC's
- Thermal Model
- Thermal Analysis
- Nodal Temps
- POD Reduction
- Struct. Model
- Thermal Loads
- Temp. Effect on Mat. Properties
- Geometric Stiffness
- Structural Stiffness
- Struct. Dynamic EOMs
- Ritz Modal Space
- Aero Loads
- Structural Deformation
- Aero ROM
- Aeroheating
- Unsteady Aero
- Vehicles Motion
- Elastic BC's
- Structural Layout
- Struct. Dynamic EOMs
Structural Dynamic Response ROM

- Structural dynamic EOMs reduced by using truncated set of reference modes
- Stiffness matrix are modified due to material degradation and thermal stress

\[ M_s \ddot{x}(t) + K_s^*(T)x(t) = F_s(t), \quad \text{where} \quad K_s^*(T) \equiv K_s(T) + K_G(T) \]

- Mode shapes change at every time step due to transient temperature distribution
- Select Ritz modes, \( \Phi_{\text{ref}} \), based on free vibration modes at a reference thermal state
- Reference modes no longer orthogonal with respect to stiffness since stiffness changes at every time step
- Reduced system of equations of motion given by: \(^1\)

\[ m_s \dddot{d}(t) + k_s^*d(t) = f_s(t), \quad \text{where} \quad k_s^*(T) = \Phi_{\text{ref}}^T K_s^*(T) \Phi_{\text{ref}} \]

\[ f_s(t) = \Phi_{\text{ref}}^T F_s(t) \]

**Fixed set of Ritz modes avoids need for eigenvalue solution at every time step**

Efficient Updating of Stiffness/Thermal Loads

- Use kriging to update generalized stiffness matrix and thermal load vector directly based on the nodal temperatures
- Simple model contains thousands of thermal DOFs → must parameterize temperature distribution in terms of smaller number of variables
- **Solution**: Use POD modal coordinates to parameterize temperature distribution

**Original Approach**

**Current Approach**

\[ m_T \ddot{c}(t) + k_T c(t) = f_T(t) \]

**POD modes provide optimal means for parameterizing temp. distribution**

Methodology for Bounding POD Modal Coordinates

Bound flight conditions ($M$, $\alpha$, $h$)

Select combinations of ($M$, $\alpha$, $h$) using LHS

Must include: ($M_{\text{max}}$, $\alpha_{\text{max}}$, $h_{\text{min}}$) and ($M_{\text{min}}$, $\alpha_{\text{min}}$, $h_{\text{max}}$)

ATE sim$_1$ → Thermal snapshots → POD basis → Max/min POD modal coords → Kriging surfaces: Bounds vs. ($M$, $\alpha$, $h$)

ATE sim$_2$ → Thermal snapshots → POD basis → Max/min POD modal coords → Kriging surfaces: Bounds vs. ($M$, $\alpha$, $h$)

ATE sim$_3$ → Thermal snapshots → POD basis → Max/min POD modal coords → Kriging surfaces: Bounds vs. ($M$, $\alpha$, $h$)

ATE sim$_4$ → Thermal snapshots → POD basis → Max/min POD modal coords → Kriging surfaces: Bounds vs. ($M$, $\alpha$, $h$)

... ATE sim$_n$ → Thermal snapshots → POD basis → Max/min POD modal coords → Kriging surfaces: Bounds vs. ($M$, $\alpha$, $h$)

Maximize/minimize for each modal coord

POD bounds
Kriging Error Assessment

- Investigated maximum $L_\infty$ error of kriging ROMs over 500 evaluation cases
- Reference solution for error calculation is based on full order model (NASTRAN)

![Graph 1: Max $L_\infty$ for Generalized Stiffness Matrix ROM Using 2nd Order Regression](image1)

Max $L_\infty$, **Generalized Stiffness Matrix** ROM Using 2nd Order Regression

![Graph 2: Max $L_\infty$ for Physical Thermal Load Vector ROM Using 2nd and 3rd Order Regressions](image2)

Max $L_\infty$, **Physical Thermal Load Vector** ROM Using 2nd and 3rd Order Regressions

- Error is higher for thermal loads due to higher order dependence on temperature
- Used 3rd order regression instead of 2nd order for thermal load vector ROM only
- **5x computational savings due to use of structural ROM**

**2nd order regression for Stiffness Matrix and 3rd order regression for Thermal Loads**